Aim: What is and how do we use Descartes’s Rule of Signs?

Do Now: 1) Consider the function: \( f(x) = 3x^3 - 5x^2 + 6x - 4 \)
   a) The polynomial has three “variations in sign.” What do you think that means?
   b) How many real zeros should this function have?

Development: 1)a) A variation in sign means that two consecutive coefficients have opposite signs. The difference in sign from one term to the next is a variation - there are three here.

b) We would expect that this should have three real zeros. Of course we’ve seen before that the zeros do not always actually have to be real. The highest degree of the polynomial tells us how many zeros there are - period - they may not necessarily be real.

There is a rule - although not a very exact rule - it can give us some insight into how many real roots a function might have and what signs they will be.

It is known as Descartes’s Rule of Signs and this is how it works:
If given a certain function with a polynomial that has real coefficients we’ll call \( f(x) \):
1) The number of positive real zeros of \( f(x) \) is either equal to the number of variations in sign of \( f(x) \) or is less than that number by an even integer.
2) The number of negative real zeros of \( f(x) \) is either equal to the number of variations in sign of \( f(-x) \) or is less than that number by an even integer.

So consider the example in the “Do Now.”
There are three variations in sign in the original function.
This means that the number of positive real roots is going to be 3 or 1.
If we find \( f(-x) = 3(-x)^3 - 5(x)^2 + 6(-x) - 4 \) we get
\[ f(-x) = -3x^3 - 5x^2 - 6x - 4. \] We see NO variations in sign.
This means there are NO negative real roots.

Applications: Find how many real roots - positive and negative - the function could have based on Descartes’s Rule of Signs:
2) \( f(x) = x^3 + 7x^2 - 5x + 19 \)
3) \( g(x) = -6x^4 + 5x^3 + 7x^2 - 8x - 2 \)
4) \( h(x) = 3x^5 + 4x^3 - 6x^2 - 10 \)
5) \( k(x) = 2x^4 - 3x^2 + 6x - 7 \)
Answers:
2) positive: 2 variations = 2 or 0 positive real roots
   \[ f(-x) = -x^3 + 7x^2 + 5x + 19 \]
   negative: 1 variation = 1 negative real root
3) positive: 2 variations = 2 or 0 positive real roots
   \[ g(-x) = -6x^4 - 5x^3 + 7x^2 + 8x - 2 \]
   negative: 2 variations = 2 or 0 negative real roots
4) positive: 1 variation = 1 positive real root
   \[ h(-x) = -3x^5 - 4x^3 - 6x^2 - 10 \]
   negative: 0 variations = 0 negative real roots
5) positive: 3 variations = 3 or 1 positive real roots
   \[ k(-x) = 2x^4 - 3x^2 - 6x - 7 \]
   negative: 1 variation = 1 negative real root

>Notice that there are “missing” terms in the last two examples. How do these
affect the count of the variations?
<They do not. Since the coefficients of these “missing” terms are actually 0
they don’t have a true sign. Therefore, they don’t matter.

Homework: HEATH: p245 #37, 48-54
5th ed HOUGHTON-MIFFLIN: p128: #42, 65-68, 69-71 (part a only)
When using 3rd ed HOUGHTON-MIFFLIN use Worksheet #24
1) Use synthetic division to show that \( x = \frac{2}{3} \) is a solution of
\[
48x^3 - 80x^2 + 41x - 6 = 0
\]

For #2-8, use Descartes' Rule of Signs to determine the possible numbers of positive and negative real zeros of \( f(x) \) or \( g(x) \):

2) \( f(x) = 2x^4 - x^3 + 6x^2 - x + 5 \)

3) \( f(x) = 3x^4 + 5x^3 - 6x^2 + 8x - 3 \)

4) \( g(x) = 4x^3 - 5x + 8 \)

5) \( g(x) = 2x^3 - 4x^2 - 5 \)

6) \( f(x) = x^3 + x^2 - 4x - 4 \)

7) \( f(x) = -3x^3 + 20x^2 - 36x + 16 \)

8) \( f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8 \)
Aim: What is and how do we use Descartes’s Rule of Signs?

**Descartes' Rule of Signs** is a method of determining the maximum number of positive and negative *real* roots of a polynomial.

Steps:
1. Polynomial function must be in descending order and have a non zero constant.
2. Find the number of positive real roots by counting the number of consecutive sign changes and then decrease this number of changes by 2. ex. if there are 3 changes then there can be 3 or 1 positive zeroes. In this example there may be a pair of complex roots.
3. Find the number of negative roots by substituting \(-x\) for each \(x\). Count the number and decrease by 2 until you no longer have a positive number. For example if there are 4 changes there may be 4, 2, or 0 real negative zeroes. Again there may be complex zeroes in this function.
4. State the number of possible real positive and negative roots.

1) \(a\) \(f(x) = x^5 - x^4 + 3x^3 + 9x^2 - x + 5\)

2) \(f(x) = x^3 + 7x^2 - 5x + 19\)

3) \(g(x) = -6x^4 + 5x^3 + 7x^2 - 8x - 2\)

4) \(h(x) = 3x^5 + 4x^3 - 6x^2 - 10\)

5) \(k(x) = 2x^4 - 3x + 6x - 7\)